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Monopole-like Excitations as a Source of Confinement in the SU(2)-Gluodynamics

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Abstract

By making use of the Abelian projection method, a dual version of the $SU(2)$-gluodynamics with manifest monopole-like excitations, arising from the integration over singular gauge transformations, is formulated in the continuum limit. The resulting effective theory emerges due to the summation over the grand canonical ensemble of these excitations in the dilute gas approximation. As a result, the dual Abelian gauge boson acquires a nonvanishing (magnetic) mass due to the Debye screening effects in such a gas. The obtained theory is then used for the construction of the corresponding effective potential of monopole loop currents and the string representation. Finally, by virtue of this representation, confining properties of the $SU(2)$-gluodynamics are emphasized.

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1 Introduction

Nowadays, it is commonly argued that the method of Abelian projection [1] is one of the most challengable approaches to solve the problem of confinement in QCD within the dynamical scheme of a dual superconductor [2] (for a recent review see *e.g.* [3]). In particular, a detailed perturbative analysis of the $SU(2)$-QCD within the dual approach has been performed [4, 5], and the asymptotic freedom of the resulting effective Abelian theory has been proved. As far as the property of confinement in the Abelian-projected $SU(N)$-QCD is concerned, it has been argued in Refs. [5, 6] that it occurs owing to the condensation of Cooper pairs of magnetic monopoles, described by the magnetic Higgs field. Spontaneous breaking of the resulting $U(1)$ symmetries then leads to the generation of the mass terms of the dual gauge fields. This makes the effective Abelian-projected $SU(N)$ gauge theory, obtained in this way, quite similar to the (London limit of the) dual Abelian Higgs type model with the $[U(1)]^{N-1}$ gauge invariance. In the latter model, confinement can be analytically studied by casting the corresponding partition function into the form of an integral over the world-sheets of the closed Abrikosov-Nielsen-Olesen strings [7] by making use of the so-called path-integral duality transformation. This transformation elaborated on for the Abelian Higgs model in Refs. [8, 9] has been employed for a derivation of the string representations for the partition functions and field strength correlators in Abelian-projected $SU(2)$- and $SU(3)$-QCD in Refs. [10] and [11], respectively [1]. After that, performing the derivative expansion [13] of the so-obtained string effective action, one gets as the first two terms of this expansion the usual Nambu-Goto term and the so-called rigidity term [14], whose coupling constants ensure confinement (in the sense of the Wilson's area law [15]) and stability of the Abrikosov-Nielsen-Olesen strings.

The aim of the present paper is to derive an effective low-energy dual theory of Abelian-projected $SU(2)$-gluodynamics in the continuum limit by summing over the grand canonical ensemble of monopole loop currents, which emerge during the Abelian projection. Moreover, in this way we shall not make the standard assumption on the formation and subsequent condensation of Cooper pairs of magnetic monopoles, but shall rather treat the ensemble of monopole loop currents in the dilute gas approximation. Next, in order to achieve our main goal, which is a manifestation of confinement in the $SU(2)$-gluodynamics, we find it necessary to derive a string representation of the obtained theory. The latter one is implied as a certain mechanism realizing the independence of the Wilson loop describing a test particle, electrically charged *w.r.t.* the maximal Abelian $U(1)$ subgroup of the original $SU(2)$ group, of the shape of some surface bounded by the contour of this Wilson loop. The construction of such a mechanism, which will be performed below, is based on the summation over branches of the multivalued effective potential of monopole loop currents, which emerges in the representation of the obtained dual model in terms of an integral over these currents. Note that such an approach is the 4D generalization of the corresponding 3D one, investigated in Ref. [16]. In that paper, it has been demonstrated that this approach parallels the one proposed in Ref. [17] for the construction of a string representation of 3D compact QED (see also Ref. [18] for the 4D generalizations). Notice also that within our approach, the dual gauge field acquires a mass dynamically, *i.e.* by virtue of the Debye screening in the gas of monopole loop currents. The appearance of this mass then leads to a nonvanishing string tension and thus confinement of an electrically charged test particle. Such a mechanism of the mass generation conceptually differs from the one of Refs. [5, 19], which employed among others the cumulant expansion theorem in the bilocal approximation [20]. As a by-product of the

[1]The evaluation of field strength correlators in Abelian-projected theories by another methods has been independently performed in Ref. [12].

present work, we propose a method of a derivation of the effective dual theory, describing a 4D dilute gas of monopole loop currents, which does not exploit the corresponding lattice partition function, as it has been done in Ref. [18].

The organization of the paper is as follows. In the next Section, we shall revisit a derivation of the effective dual theory, corresponding to the Abelian-projected $SU(2)$-gluodynamics. After that, we shall perform the path-integral summation over the grand canonical ensemble of fluctuating random monopole loop currents, which emerge during the Abelian projection, in the dilute gas approximation and arrive at a certain effective field theory describing this ensemble. In Section 3, this theory will be used for the calculation of the potential of monopole loop currents and the derivation of the corresponding string representation. Finally, the latter one will yield us confinement of an electrically charged test particle in the sense of the Wilson's area law. The résumé of the work and concluding remarks are presented in Summary and Discussions.

2 Effective Dual Theory of the Abelian-Projected $SU(2)$-Gluodynamics

In the present Section, we shall derive an effective dual model corresponding to the Abelian-projected $SU(2)$-gluodynamics. The starting points of this derivation will somewhat parallel that of Refs. [4, 5, 6]. The action under study reads [2]

$$S_{\rm YM}\left[A_\mu^i\right]=\frac12{\rm tr}\int d^4xF_{\mu\nu}^2, \tag{1}$$

where $F_{\mu\nu}=F_{\mu\nu}^iT^i$ with $F_{\mu\nu}^i=\partial_\mu A_\nu^i-\partial_\nu A_\mu^i+g\varepsilon^{ijk}A_\mu^jA_\nu^k$ and $T^i=\frac{\tau^i}{2}$, $i=1,2,3$. Here, τ^i's stand for Pauli matrices, and g is the QCD ("electric") coupling constant.

One can perform the gauge transformation $A'_\mu=UA_\mu U^\dagger+\frac{i}{g}U\partial_\mu U^\dagger$, so that the gauge-transformed field A'_μ obeys the so-called maximal Abelian gauge fixing condition (see *e.g.* Refs. [4, 5]) $\left(\partial_\mu\pm iga'_\mu\right)\left(A'^1_\mu\pm iA'^2_\mu\right)=0$, where $a'_\mu\equiv A'^3_\mu$. Notice that the maximal Abelian gauge fixing condition can be written as follows $\mathcal{D}'^{ab}_\mu A'^b_\mu=0$, where $\mathcal{D}'^{ab}_\mu=\partial_\mu\delta^{ab}-g\varepsilon^{ab3}a'_\mu$. Once being rewritten in this form, this gauge can be easily recognized as the standard background gauge [21] with the field a'_μ playing the rôle of the background [3]. The gauge transformed field strength tensor then reads $F'_{\mu\nu}=U\left(F_{\mu\nu}+F_{\mu\nu}^{\rm sing.}\right)U^\dagger$, where the singular contribution has the form $F_{\mu\nu}^{\rm sing.}=\frac{i}{g}\left(\left[\partial_\mu,\partial_\nu\right]U^\dagger\right)U$. This contribution comes about from the singular character of the matrix U of the gauge transformation [1, 3, 4, 5, 23] and describes world-sheets of the Dirac strings. Clearly, integration over all possible singular gauge transformations results to an integration over $F_{\mu\nu}^{\rm sing.}$.

Let us next single out the diagonal (neutral) component $a_\mu\equiv A_\mu^3$ of the field A_μ by making use of the decomposition [4] $A_\mu=a_\mu T^3+A_\mu^aT^a\equiv\mathcal{A}_\mu+\mathcal{C}_\mu$, where $a=1,2$. Consequently, one has for the field strength tensor

[2]Throughout the present paper, we work in the Euclidean space-time.

[3]Recently, in Ref. [22] this analogue between the two gauges has been employed for the investigation of the Wilsonian exact renormalization group flow of gluodynamics in the maximal Abelian gauge.

[4]From now on, we omit for brevity the prime denoting the gauge transformed fields, implying everywhere the maximal Abelian gauge fixing condition.

$$F_{\mu\nu} \equiv F_{\mu\nu}\left[\mathcal{A}+\mathcal{C}\right] = F_{\mu\nu}\left[\mathcal{A}\right] + \left(D\left[\mathcal{A}\right]\wedge\mathcal{C}\right)_{\mu\nu} - ig\left[\mathcal{C}_\mu,\mathcal{C}_\nu\right], \tag{2}$$

where $\left(\mathcal{O}\wedge\mathcal{G}\right)_{\mu\nu} \equiv \mathcal{O}_\mu\mathcal{G}_\nu - \mathcal{O}_\nu\mathcal{G}_\mu$, and $D_\mu\left[\mathcal{A}\right] = \partial_\mu - ig\left[\mathcal{A}_\mu,\cdot\right]$. Eq. (2) can be straightforwardly rewritten as follows $F_{\mu\nu} = \left(f_{\mu\nu}+C_{\mu\nu}\right)T^3 + S^a_{\mu\nu}T^a$. Here, $f_{\mu\nu} = \left(\partial\wedge a\right)_{\mu\nu}$ and $C_{\mu\nu} = g\varepsilon^{ab3}A^a_\mu A^b_\nu$ stand for the contributions of diagonal and off-diagonal components of the gluon field to the diagonal part of the field strength tensor, respectively, and $S^a_{\mu\nu} = \left(\mathcal{D}^{ab}\wedge A^b\right)_{\mu\nu}$ is the off-diagonal part of the field strength tensor. This yields the following decomposition of the action (1) (taken now on the gauge transformed fields)

$$S_{\rm YM}\left[A^i_\mu\right] = \frac14\int d^4x\left(f_{\mu\nu}+C_{\mu\nu}+\left(F^{\rm sing.}_{\mu\nu}\right)^3\right)^2 + \frac14\int d^4x\left(S^a_{\mu\nu}+\left(F^{\rm sing.}_{\mu\nu}\right)^a\right)^2, \tag{3}$$

where $\left(F^{\rm sing.}_{\mu\nu}\right)^i = 2\,{\rm tr}\left(T^iF^{\rm sing.}_{\mu\nu}\right)$. It is worth remarking that the non-Abelian commutator term $C_{\mu\nu}$, when evaluated with the singular part of the gauge transformed field, $A^{\rm sing.}_\mu = \frac{i}{g}U\partial_\mu U^\dagger$, generates among others monopole contributions. Such monopole terms have, however, been shown to become cancelled by the corresponding terms arising during the evaluation of the Abelian field strength $f_{\mu\nu}$ at $\left(A^{\rm sing.}_\mu\right)^3$ [5, 23]. This leaves in Eq. (3), besides the contributions of the non-singular gauge field configurations to be treated as quantum fluctuations, only the (singular) contributions of Dirac strings.

As it has been demonstrated in Refs. [4, 5], all the terms on the R.H.S. of Eq. (3) depending on the off-diagonal gluons A^a_μ's contribute to the momentum dependence of the running coupling constant and yield asymptotic freedom. Namely, the running coupling constant coincides with that of the original gluodynamics and reads $g(\mu)^{-2} = g(\mu_0)^{-2} + \frac{b_0}{8\pi^2}\ln\frac{\mu}{\mu_0}$, where $b_0 = \frac{11C_2(G)}{3}$ with $C_2(G)$ standing for the Casimir operator of the adjoint representation of the group $G = SU(2)$ under consideration, *i.e.* $C_2(G) = 2$. Since in what follows our aim will be the investigation of the confining (*i.e.* infrared) properties of the Abelian-projected $SU(2)$-gluodynamics (rather than the problems of its renormalization, related to the region of asymptotic freedom), we shall disregard the A^a_μ-dependent terms (This approximation is usually referred to as the Abelian dominance hypothesis [24].). Within this approximation, the resulting effective action takes the form

$$S_{\rm eff.}\left[a_\mu,\mathcal{F}_{\mu\nu}\right] = \frac14\int d^4x\left(f_{\mu\nu}+\mathcal{F}_{\mu\nu}\right)^2, \tag{4}$$

where we have denoted for brevity $\mathcal{F}_{\mu\nu} \equiv \left(F^{\rm sing.}_{\mu\nu}\right)^3$.

The monopole current is defined via the modified Bianchi identities as follows

$$j^M_\nu = \partial_\mu\left(\tilde f_{\mu\nu}+\tilde{\mathcal{F}}_{\mu\nu}\right) = \frac12\varepsilon_{\mu\nu\lambda\rho}\partial_\mu\mathcal{F}_{\lambda\rho} \tag{5}$$

with $\tilde f_{\mu\nu} = \frac12\varepsilon_{\mu\nu\lambda\rho}f_{\lambda\rho}$, *etc.* Thus, in what follows we shall regard the obtained effective theory (4) as a $U(1)$ gauge theory with monopole loop currents. Our aim then will be to investigate confining properties of such a theory by a derivation of its string representation. To this end, let us first cast the partition function under study, $\mathcal{Z} = \int\mathcal{D}\mathcal{F}_{\mu\nu}\mathcal{D}a_\mu\exp\left(-S_{\rm eff.}\left[a_\mu,\mathcal{F}_{\mu\nu}\right]\right)$, to the dual form [5]. This can be done by making use of the first-order formalism, *i.e.* linearizing the square $f^2_{\mu\nu}$ in Eq. (4) by introducing an integration over an auxiliary antisymmetric tensor field $b_{\mu\nu}$ as follows

[5]Notice that the gauge fixing term of the Abelian field is assumed to be included into the integration measure $\mathcal{D}a_\mu$.

$$\mathcal{Z} = \int \mathcal{D}\mathcal{F}_{\mu\nu}\mathcal{D}a_\mu\mathcal{D}b_{\mu\nu}\exp\left\{-\int d^4x\left[\frac14 b_{\mu\nu}^2 + \frac{i}{2}\tilde{b}_{\mu\nu}f_{\mu\nu} + \frac12 f_{\mu\nu}\mathcal{F}_{\mu\nu} + \frac14\mathcal{F}_{\mu\nu}^2\right]\right\}. \tag{6}$$

Integration over the a_μ-field leads to the constraint $\partial_\mu\left(\tilde{b}_{\mu\nu} - i\mathcal{F}_{\mu\nu}\right) = 0$, whose resolution yields $b_{\mu\nu} = i\tilde{\mathcal{F}}_{\mu\nu} + (\partial\wedge b)_{\mu\nu}$, where b_μ is now the "magnetic" potential dual to the "electric" potential a_μ. Substituting this representation for $b_{\mu\nu}$ into Eq. (6), we get

$$\mathcal{Z} = \left\langle\int \mathcal{D}b_\mu \exp\left\{-\int d^4x\left[\frac14 b_{\mu\nu}^2 - ib_\mu j_\mu^M\right]\right\}\right\rangle_{j_\mu^M}, \tag{7}$$

where from now on, $b_{\mu\nu}$ denotes simply $(\partial\wedge b)_{\mu\nu}$. In Eq. (7), the integration over $\mathcal{F}_{\mu\nu}$'s has transformed to a certain average over monopole loop currents, $\langle\ldots\rangle_{j_\mu^M}$, whose concrete form will be specified below.

It is worth noting that due to the conservation of the monopole current j_μ^M, the dual action standing in the exponent on the R.H.S. of Eq. (7) is invariant under the magnetic gauge transformations $b_\mu \to b_\mu + \partial_\mu\chi$. Again, we shall imply that the gauge fixing term for the b_μ-field is included into the integration measure $\mathcal{D}b_\mu$. Moreover, we shall specify the gauge to be the Fock-Schwinger one, *i.e.* $x_\mu b_\mu(x) = 0$.

Our next aim is to sum up over the ensemble of monopole loop currents in the dual theory (7). To this end, we shall treat this ensemble as the grand canonical one and make an assumption that monopole loop currents form a dilute gas. Then, since the energy of a single monopole is known to be a quadratic function of its flux, it is more energetically favorable for the vacuum to support a configuration of two monopoles of a unit magnetic charge than one monopole of the double charge. Therefore, only the monopoles with the minimal charges $q_a g_m$ with $q_a = \pm 1$ are essential, whereas the ones with $|q_a| > 1$ tend to dissociate into those with $|q_a| = 1$. Here, the magnetic coupling constant g_m is related to the QCD coupling g via the topological quantization condition $gg_m = 4\pi n$. In what follows, we shall set in this condition $n = 1$, which parallels the above restriction to the monopoles possessing the minimal charge only. Obviously, the same restriction then holds for the Dirac strings ending up at monopole-antimonopole pairs, as well. The collective current of N monopoles takes the form

$$j_\mu^{M\,(N)}(x) = \frac{4\pi}{g}\sum_{a=1}^{N} q_a \oint dz_\mu^a \delta\left(x - x^a(\tau)\right), \tag{8}$$

where the a-th monopole loop current is parametrized by the vector $x_\mu^a(\tau) = y_\mu^a + z_\mu^a(\tau)$, $0 \le \tau \le 1$. Here, $y_\mu^a = \int\limits_0^1 d\tau x_\mu^a(\tau)$ denotes the position of the a-th loop current, whereas the vector $z_\mu^a(\tau)$ corresponds to its shape, both of which should be averaged over independently in $\langle\ldots\rangle_{j_\mu^M}$ [6]. Namely, the average $\langle\ldots\rangle_{j_\mu^M}$ with the collective current (8) takes the form

$$\left\langle\mathcal{O}\left[j_\mu^M\right]\right\rangle_{j_\mu^M} = \prod_{i=1}^{N}\int d^4y^i\mathcal{D}z^i\mu\left[z^i\right]\sum_{q_a=\pm1}\mathcal{O}\left[j_\mu^{M\,(N)}\right]. \tag{9}$$

[6] As it follows from Eq. (5), $\mathcal{F}_{\mu\nu}$ corresponding to the current (8) is nothing else, but the field strength tensor of N Dirac strings, $\mathcal{F}_{\mu\nu}(x) = -\frac{4\pi}{g}\frac12\varepsilon_{\mu\nu\lambda\rho}\sum\limits_{a=1}^{N} q_a\int d\sigma_{\lambda\rho}\left(x^a(\xi)\right)\delta\left(x - x^a(\xi)\right)$. Here, $x^a(\xi)$ is a vector parametrizing the world-sheet of the a-th string with ξ standing for the two-dimensional coordinate.

Here, $\mu\left[z^i\right]$ is a certain rotation- and translation invariant integration measure over the shapes of monopole loop currents, whose concrete form will not be specified here (For example, one can take it in the form of the properly normalized measure of an ensemble of oriented random loops, representing trajectories of scalar particles,

$$\int \mathcal{D}z^i \mu\left[z^i\right] \mathcal{O}\left[z^i\right] = \mathcal{N}\int\limits_0^{+\infty}\frac{ds_i}{s_i}\int\limits_{u(0)=u(s_i)}\mathcal{D}u\left(s_i'\right)\exp\left(-\frac{1}{4}\int\limits_0^{s_i}\dot{u}^2\left(s_i'\right)ds_i'\right)\mathcal{O}\left[u\left(s_i'\right)\right],$$

where the vector $u_\mu\left(s_i'\right)$ parametrizes the same contour as the vector $z_\mu^i(\tau)$.).

One can now write down the contribution of N monopole loop currents to the partition function of their grand canonical ensemble. Owing to Eqs. (8) and (9) it reads

$$\mathcal{Z}^M\left[b_\mu\right] = 1 + \sum_{N=1}^{\infty}\frac{\zeta^N}{N!}\left\langle \exp\left(i\int d^4x b_\mu j_\mu^M\right)\right\rangle_{j_\mu^M} =$$

$$= 1 + \sum_{N=1}^{\infty}\frac{(2\zeta)^N}{N!}\left\{\int d^4y\int \mathcal{D}z\mu[z]\cos\left(\frac{4\pi}{g}\oint dz_\mu b_\mu(x)\right)\right\}^N. \tag{10}$$

Here, $\zeta \propto \mathrm{e}^{-S_0}$ is the so-called fugacity term (Boltzmann factor of a single monopole loop current) of dimension $(\mathrm{mass})^4$ with the action of a single loop current given by $S_0 = \mathrm{const.}g_m^2$.

In order to evaluate the path-integral over z_μ's in Eq. (10), let us employ the above mentioned dilute gas approximation, which requires that typical distances between monopole loop currents are much larger than their sizes. This means that generally $|y^a| \gg |z^a|$, where from now on $|y| \equiv \sqrt{y_\mu^2}$. Let us denote characteristic distances $|y|$ by L, characteristic sizes of monopole loop currents $\left(=\int\limits_0^1 d\tau\sqrt{\dot{z}^2}\right)$ by a, and perform the Taylor expansion of $b_\mu(x)$ up to the first order in a/L (which is the first one yielding a nonvanishing contribution to the integral $\oint dz_\mu b_\mu(x)$ on the R.H.S. of Eq. (10)),

$$b_\mu(x) = b_\mu(y) + L^{-1}z_\nu n_\nu b_\mu(y) + O\left(\left(\frac{a}{L}\right)^2\right). \tag{11}$$

Here, we have denoted $n_\nu = \frac{y_\nu}{|y|}$ and estimated the derivative $\partial/\partial y_\nu$ as n_ν/L. Then, the substitution of expansion (11) into Eq. (10) yields

$$\int \mathcal{D}z\mu[z]\cos\left(\frac{4\pi}{g}\oint dz_\mu b_\mu(x)\right) \simeq \int \mathcal{D}z\mu[z]\cos\left(\frac{4\pi}{gL}n_\nu b_\mu(y)\mathcal{P}_{\mu\nu}[z]\right) =$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(\frac{4\pi}{gL}\right)^{2n} n_{\nu_1}b_{\mu_1}(y)\cdots n_{\nu_{2n}}b_{\mu_{2n}}(y)\int \mathcal{D}z\mu[z]\mathcal{P}_{\mu_1\nu_1}[z]\cdots\mathcal{P}_{\mu_{2n}\nu_{2n}}[z], \tag{12}$$

where $\mathcal{P}_{\mu\nu}[z] \equiv \oint dz_\mu z_\nu$ stands for the tensor area associated with the contour parametrized by $z_\mu(\tau)$ [7]. Due to the rotation- and translation invariance of the measure $\mu[z]$, the average of the product of the tensor areas can be written in the form

[7] One can check that for the plane contour, $\mathcal{P}_{\mu\nu} = -\mathcal{P}_{\nu\mu} = -S$, $\mu < \nu$, where S is the area inside the contour.

$$\int \mathcal{D}z\mu[z]\mathcal{P}_{\mu_1\nu_1}[z]\cdots\mathcal{P}_{\mu_{2n}\nu_{2n}}[z] = \frac{\left(a^2\right)^{2n}}{(2n-1)!!}\left[\hat{1}_{\mu_1\nu_1,\mu_2\nu_2}\cdots\hat{1}_{\mu_{2n-1}\nu_{2n-1},\mu_{2n}\nu_{2n}} + \text{permutations}\right]. \tag{13}$$

Here $\hat{1}_{\mu\nu,\lambda\rho} = \frac{1}{2}\left(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}\right)$, and the normalization factor $(2n-1)!!$ is explicitly extracted out since the sum in square brackets on the R.H.S. of Eq. (13) contains $(2n-1)!!$ terms. Substituting now Eq. (13) into Eq. (12), recalling that we have adopted for the field b_μ the Fock-Schwinger gauge, so that $n_\mu b_\mu(y) = 0$ [8], and denoting $\frac{2\sqrt{2}\pi a^2}{gL}$ $(\ll a)$ by Λ^{-1}, where Λ acts as a natural UV momentum cutoff, we finally obtain

$$\int \mathcal{D}z\mu[z]\cos\left(\frac{4\pi}{g}\oint dz_\mu b_\mu(x)\right) \simeq \cos\left(\frac{|b_\mu(y)|}{\Lambda}\right).$$

Owing to this result, the partition function of the grand canonical ensemble of monopole loop currents reads $\mathcal{Z}^M\left[b_\mu\right] = \exp\left[2\zeta\int d^4x\cos\left(\frac{|b_\mu|}{\Lambda}\right)\right]$. Together with Eq. (7), it yields the desired expression for the partition function of an effective dual theory of the Abelian-projected $SU(2)$-gluodynamics, which has the form

$$\mathcal{Z} = \int \mathcal{D}b_\mu \exp\left\{-\int d^4x\left[\frac{1}{4}b_{\mu\nu}^2 - 2\zeta\cos\left(\frac{|b_\mu|}{\Lambda}\right)\right]\right\}. \tag{14}$$

The ("magnetic") Debye mass of the b_μ-field, which it acquires due to the screening by magnetic loop currents, can now be immediately read off from the expansion of the cosine and has the form $m = \frac{\sqrt{2\zeta}}{\Lambda}$. Notice also that a partition function of the type (14) (considered *ad hoc* as a continuum version of the corresponding lattice expression) has been used in Ref. [18] as a starting point for the construction of the string representation of the 4D compact QED. Our construction of an analogous representation for the model (14) will be performed in a more simple way. Namely, we shall construct such a string representation by virtue of the representation of the model under study in terms of the monopole loop currents, which is a 4D generalization of the corresponding expression for the 3D partition function in terms of the monopole densities, investigated in Ref. [16].

3 String Representation and Confinement

In the present Section, we shall construct the string representation for the Wilson loop of an electrically charged (*w.r.t.* the maximal Abelian $U(1)$ subgroup of the original $SU(2)$ group) test particle in the effective Abelian-projected theory (14). Such a representation will enable us to manifest confinement in this theory. To get the desired string representation, we shall first derive the representation of the corresponding partition function in terms of the integral over monopole loop currents. To this end, notice that integrating over the field b_μ in Eq. (7) one gets for the statistical weight $Z\left[j_\mu^M\right]$ defined by the relation $\mathcal{Z} \equiv \left\langle Z\left[j_\mu^M\right]\right\rangle_{j_\mu^M}$ an expression in the form of the Coulomb interaction between the monopole loop currents,

[8] Within the dilute gas approximation, where $\frac{\partial}{\partial y_\nu} \to \frac{n_\nu}{L}$, the Fock-Schwinger gauge is equivalent to the Lorentz one.

$$Z\left[j_\mu^M\right]=\exp\left(-\frac{1}{8\pi^2}\int d^4xd^4x'j_\mu^M(x)\frac{1}{(x-x')^2}j_\mu^M(x')\right).$$

Owing to this equation, one has [9]

$$\mathcal{Z}=1+\sum_{N=1}^{\infty}\frac{\zeta^N}{N!}\left\langle\int\mathcal{D}j_\mu\delta\left(j_\mu-j_\mu^M\right)Z\left[j_\mu\right]\right\rangle_{j_\mu^M}=$$

$$=\int\mathcal{D}j_\mu\mathcal{D}\lambda_\mu\exp\left[-\frac{1}{8\pi^2}\int d^4xd^4x'j_\mu(x)\frac{1}{(x-x')^2}j_\mu(x')-i\int d^4x\lambda_\mu j_\mu+2\zeta\int d^4x\cos\left(\frac{|\lambda_\mu|}{\Lambda}\right)\right], \tag{15}$$

where the term fixing the Fock-Schwinger gauge for the Lagrange multiplier λ_μ is again assumed to be included into the integration measure. Notice that $\mathcal{D}j_\mu$ here is the standard integration measure over the vector field, which is of the same form as $\mathcal{D}\lambda_\mu$.

Clearly, due to the δ-function standing on the R.H.S. of the first equality in Eq. (15), if we integrate the Lagrange multiplier λ_μ out of this equation, the resulting expression will be just the desired representation of the partition function in terms of the monopole loop currents. In order to carry out such an integration, one should solve the saddle-point equation

$$\frac{\lambda_\mu}{|\lambda_\mu|}\sin\left(\frac{|\lambda_\mu|}{\Lambda}\right)=-\frac{i\Lambda}{2\zeta}j_\mu. \tag{16}$$

This can be done by noting that its L.H.S. is a vector in the direction λ_μ, which means that it can be equal to the R.H.S. only provided that the direction of the vector λ_μ coincides with the direction of the vector j_μ. Therefore, it is reasonable to seek for a solution to Eq. (16) in the form $\lambda_\mu=|\lambda_\mu|\frac{j_\mu}{|j_\mu|}$. Then, Eq. (16) reduces to the scalar equation $\sin\left(\frac{|\lambda_\mu|}{\Lambda}\right)=-\frac{i\Lambda|j_\mu|}{2\zeta}$. Straightforward solution of the latter one yields the desired representation for the partition function

$$\mathcal{Z}=\int Dj_\mu\exp\left\{-\left[\frac{1}{8\pi^2}\int d^4xd^4x'j_\mu(x)\frac{1}{(x-x')^2}j_\mu(x')+V\left[j_\mu\right]\right]\right\}, \tag{17}$$

where the complex-valued effective potential of monopole loop currents reads

$$V\left[j_\mu\right]=$$

$$=\sum_{n=-\infty}^{+\infty}\int d^4x\left\{\Lambda\left|j_\mu\right|\left[\ln\left[\frac{\Lambda}{2\zeta}\left|j_\mu\right|+\sqrt{1+\left(\frac{\Lambda}{2\zeta}\left|j_\mu\right|\right)^2}\right]+2\pi in\right]-2\zeta\sqrt{1+\left(\frac{\Lambda}{2\zeta}\left|j_\mu\right|\right)^2}\right\}. \tag{18}$$

Let us now proceed with the string representation of the Wilson loop in the effective theory (14). Assuming for a while that the monopole loop currents are absent, one has for this object the following expression

[9] In the thermodynamic limit, where the number of monopole loop currents N and the four-volume of observation V infinitely increase with the density of the loop currents $\rho=N/V$ being kept fixed, the collective current (8) can be treated as a continuous function of disorder type. In what follows, we assume that the "free path length" L of monopole loop currents is much smaller than the characteristic size of the Wilson loop of an external electrically charged test particle.

$$\langle W(C)\rangle_{a_\mu}=\left\langle\frac{1}{2}\mathrm{tr}\,P\exp\left(ig\oint\limits_C dx_\mu a_\mu T^3\right)\right\rangle_{a_\mu}\;,\;\text{where}\;\langle\ldots\rangle_{a_\mu}=\frac{\int\mathcal{D}a_\mu\,(\ldots)\exp\left(-\frac{1}{4}\int d^4x f_{\mu\nu}^2\right)}{\int\mathcal{D}a_\mu\exp\left(-\frac{1}{4}\int d^4x f_{\mu\nu}^2\right)}.$$

Next, the P-ordering can be omitted, since all the matrices commute with each other, after which we obtain

$$\langle W(C)\rangle_{a_\mu}=\left\langle\cos\left(\frac{g}{2}\oint\limits_C dx_\mu a_\mu\right)\right\rangle_{a_\mu}=\left\langle\exp\left(\frac{ig}{2}\oint\limits_C dx_\mu a_\mu\right)\right\rangle_{a_\mu}=$$

$$=\exp\left(-\frac{g^2}{32\pi^2}\oint\limits_C dx_\mu\oint\limits_C dy_\mu\frac{1}{(x-y)^2}\right),$$

which is the standard "perimeter" (Gaussian) contribution to the Wilson loop.

However in the presence of monopole loop currents, one should properly extend the field strength tensor $f_{\mu\nu}$ in analogue to Eqs. (4) and (5) in order to satisfy Bianchi identities modified by the current j_μ. This can be done by using the complete field strength tensor $f_{\mu\nu}+h_{\mu\nu}$, where the fluctuating antisymmetric tensor-disorder field $h_{\mu\nu}$ (the so-called Kalb-Ramond field [25]) just obeys these modified identities, *i.e.* $\partial_\mu\tilde{h}_{\mu\nu}=j_\nu$.

By virtue of the Stokes theorem, we then obtain for the full Wilson loop the following expression

$$\langle W(C)\rangle=\left\langle\exp\left[\frac{ig}{4}\int\limits_\Sigma d\sigma_{\mu\nu}\left(f_{\mu\nu}+h_{\mu\nu}\right)\right]\right\rangle_{a_\mu,j_\mu}=\langle W(C)\rangle_{a_\mu}\left\langle\exp\left(\frac{ig}{4}\int\limits_\Sigma d\sigma_{\mu\nu}h_{\mu\nu}\right)\right\rangle_{j_\mu}. \tag{19}$$

Here, the average over currents is defined by the partition function (17), and Σ is an arbitrary surface bounded by the contour C. Expressing $h_{\mu\nu}$ via j_μ, we can rewrite the last average on the R.H.S. of Eq. (19) directly as

$$\left\langle\exp\left(-\frac{ig}{2}\int d^4x j_\mu\eta_\mu\right)\right\rangle_{j_\mu}. \tag{20}$$

Here,

$$\eta_\mu(x)=\frac{1}{8\pi^2}\varepsilon_{\mu\nu\lambda\rho}\frac{\partial}{\partial x_\nu}\int\limits_\Sigma d\sigma_{\lambda\rho}(x(\xi))\frac{1}{(x-x(\xi))^2} \tag{21}$$

stands for the 4D solid angle, under which the surface Σ shows up to an observer located at the point x with $\xi=(\xi^1,\xi^2)$ denoting the 2D coordinate (If Σ is a closed surface surrounding the point x than by virtue of the Gauss law, $d\tilde{\sigma}_{\mu\nu}\to dS_\mu\partial_\nu-dS_\nu\partial_\mu$, one can check that for the conserved current j_μ, $\int d^4xj_\mu\eta_\mu=\int dS_\mu j_\mu$, as it should be. Here, dS_μ stands for the oriented element of the hypersurface bounded by Σ.). An apparent Σ-dependence of Eq. (20) actually drops out due to the summation over branches of the multivalued potential (18). This is the essence of the string representation of the Wilson loop in the effective dual theory (14).

Let us now consider the weak-field limit, *i.e.* the limit $\Lambda\,|j_\mu|\ll\zeta$, and investigate the (stable) minimum of the real branch of the potential of monopole loop currents. This corresponds to extracting the term with $n=0$ from the whole sum in Eq. (18). Then, since we have restricted ourselves to the only one branch of the potential, the Σ-independence of the Wilson loop is spoiled. In order to restore it, let us choose Σ to be the surface of the minimal area for a given contour C, unambiguously defined by this contour (see discussion in Ref. [16]), $\Sigma = \Sigma_{\rm min.}[C]$. Then the Wilson loop takes the form

$$\langle W(C)\rangle_{\rm weak-field} = \langle W(C)\rangle_{a_\mu} \times$$

$$\times \int \mathcal{D}j_\mu \exp\left\{-\left[\frac{1}{8\pi^2}\int d^4x d^4x' j_\mu(x)\frac{1}{(x-x')^2}j_\mu(x') + \frac{\Lambda^2}{4\zeta}\int d^4x j_\mu^2 + \frac{ig}{2}\int d^4x j_\mu \eta_\mu\right]\right\}, \tag{22}$$

where now η_μ is defined by Eq. (21) with the replacement $\Sigma \to \Sigma_{\rm min.}$. Recalling the expression for j_μ via $h_{\mu\nu}$, Eq. (22) can be written as follows

$$\langle W(C)\rangle_{\rm weak-field} = \langle W(C)\rangle_{a_\mu}\int \mathcal{D}h_{\mu\nu}\exp\left[-\int d^4x\left(\frac{\Lambda^2}{24\zeta}H_{\mu\nu\lambda}^2 + \frac{1}{4}h_{\mu\nu}^2\right) + \frac{ig}{4}\int\limits_{\Sigma_{\rm min.}} d\sigma_{\mu\nu}h_{\mu\nu}\right], \tag{23}$$

where $H_{\mu\nu\lambda} = \partial_\mu h_{\nu\lambda} + \partial_\lambda h_{\mu\nu} + \partial_\nu h_{\lambda\mu}$ is the field strength tensor of the Kalb-Ramond field $h_{\mu\nu}$. It is worth noting, that the mass of the Kalb-Ramond field following from the quadratic part of the action standing in the exponent on the R.H.S. of Eq. (23) is equal to the Debye mass m of the field b_μ following from Eq. (14). Integration over the Kalb-Ramond field is now straightforward and can be performed along the lines of Ref. [10]. Obviously, after such an integration, one gets the string effective action $S_{\rm str.} = -\ln\langle W(C)\rangle_{\rm weak-field}$ in the form of an interaction between two world-sheet elements mediated by the propagator of this field. A certain part of this interaction can be rewritten by the Stokes theorem as the "perimeter" Yukawa type interaction (see Ref. [10] for details). The remaining part, once being expanded in powers of the derivatives *w.r.t.* ξ^a's (which is equivalent to the $1/m$-expansion) by virtue of the results of Ref. [13], yields as the first two terms of this expansion the standard Nambu-Goto one and the so-called rigidity term [14], *i.e.*

$$S_{\rm str.} \simeq \sigma\int d^2\xi\sqrt{\hat g} + \frac{1}{\alpha_0}\int d^2\xi\sqrt{\hat g}\hat g^{ab}\left(\partial_a t_{\mu\nu}\right)\left(\partial_b t_{\mu\nu}\right). \tag{24}$$

Here, $\partial_a = \partial/\partial\xi^a$, $\hat g = \det\left\|\hat g^{ab}\right\|$ with $\hat g^{ab} = (\partial^a x_\mu(\xi))(\partial^b x_\mu(\xi))$ being the induced metric tensor of the world-sheet, and $t_{\mu\nu} = \frac{\varepsilon^{ab}}{\sqrt{\hat g}}\left(\partial_a x_\mu(\xi)\right)\left(\partial_b x_\nu(\xi)\right)$ standing for the so-called extrinsic curvature tensor. The string tension σ of the Nambu-Goto term (*i.e.* the coefficient in the Wilson's area law) and the inverse coupling constant of the rigidity term, $1/\alpha_0$, are completely determined via the parameters of the model (14) and read $\sigma \simeq \frac{g^2\zeta}{8\pi\Lambda^2}\ln\frac{1}{c}$ and $\frac{1}{\alpha_0} = -\frac{g^2}{128\pi}$. Here, c stands for a characteristic small dimensionless parameter, which in the model under study is reasonable to be set $c \sim g\zeta^{1/4}/\Lambda$. Notice that the string tension is obviously proportional to the square of the Debye mass m of the dual gauge field b_μ and consequently nonanalytic in the QCD coupling constant g. This result reflects the nonperturbative nature of confinement in the effective Abelian-projected theory (14) similar to that in the original non-Abelian $SU(2)$-gluodynamics.

In conclusion of this Section, note that the signs of the string tension and coupling constant of the rigidity term support the stability of strings described by the effective action $S_{\rm eff.}$. While the requirement of positiveness of the string tension is obvious already for the very existence of strings, it is worth briefly discussing the requirement of the negativeness of the coupling constant of the rigidity term. A simple argument in favour of this observation can be obtained by considering the propagator corresponding to the string effective action (24) in the so-called conformal gauge for the induced metric, $\hat{g}^{ab} = \frac{\delta^{ab}}{\sqrt{\hat{g}}}$. For a certain Lorentz index λ it reads

$$\langle x_\lambda(\xi) x_\lambda(0) \rangle = \int \frac{d^2p}{(2\pi)^2} \frac{{\rm e}^{ip^a \xi_a}}{\sigma p^2 - \frac{1}{\alpha_0}(p^2)^2}.$$

For negative α_0, this integral is well defined and is equal to

$$\langle x_\lambda(\xi) x_\lambda(0) \rangle = -\frac{1}{2\pi\sigma} \left[\ln(\mu|\xi|) + K_0 \left(\sqrt{|\alpha_0|\,\sigma} |\xi| \right) \right],$$

where μ denotes the IR momentum cutoff, and K_0 is the modified Bessel function. Contrary to that, for positive α_0 an unphysical pole in the propagator occurs, which confirms our statement.

Thus we conclude that the obtained string characteristica manifest confinement and provide us with the necessary condition for the stability of strings in the obtained effective Abelian-projected theory (14).

4 Summary and Discussions

In the present paper, by considering a grand canonical ensemble of fluctuating monopole-like excitations emerging in the Abelian-projected $SU(2)$-gluodynamics, we have derived an effective disorder field theory describing this ensemble in the continuum limit. Contrary to the previous approaches, this has been done without an assumption on the formation and condensation of Cooper pairs of monopoles, *i.e.* without introducing the corresponding magnetic Higgs field. Instead of that, we have dealt directly with the dilute Coulomb gas of monopole loop currents (describing the creation and annihilation of the monopole-antimonopole pairs). The proposed approach provided us with a natural dynamical mechanism of generation of a mass of the dual gauge field, which is due to the Debye screening in the gas of monopole loop currents.

Next, within the obtained theory we have investigated the string representation of the Wilson loop, which describes an external particle electrically charged *w.r.t.* the maximal Abelian $U(1)$-subgroup of the original $SU(2)$-group. The essence of this representation is a certain mechanism realizing the independence of the Wilson loop of some surface, bounded by its contour. As it has been illustrated, this mechanism is based on the summation over branches of the multivalued effective potential of monopole loop currents. Finally, in the weak-field limit of the obtained effective Abelian-projected theory, we have derived the string tension of the Nambu-Goto term and the inverse coupling constant of the rigidity term, which in a manifest way express confinement in the sense of the Wilson's area law and signal the stability of strings. In particular, the string tension turned out to be nonanalytic in the QCD coupling constant analogously to what happens in the original gluodynamics.

In conclusion, it is worth mentioning that the approach to the problem of confinement, investigated in the present work, essentially employed the disorder (stochastic) nature of fluctuations of monopole loop currents and related Dirac strings. In particular, the resulting stochastic correlations of topological monopole-like excitations are an important dynamical ingredient for getting the Wilson's area law. Note that this dynamical mechanism confines (chromo)electric charges of both test quarks and gluons. In this sense, the above disorder (stochastic) approach differs from the standard one, where the area law follows from the nontrivial linking of some topological objects (like Z_N-vortices [26]) with the contour of the Wilson loop. Notice that if one had fixed the gauge further, leaving, for example, the Z_N center symmetry, the gluons would have been uncharged, and the physics of their confinement would be left obscure [27]. Note also that, contrary to the most well known monopoles, Abelian-projected monopoles are described by stochastic loop currents. Owing to that, associated stochastically distributed Abrikosov-Nielsen-Olesen type strings neither do not need to be a solution of the usual classical field equations of motion nor to be described by a vacuum expectation value of some field.

Clearly, it is now a challenge to apply the present approach to the more realistic case of $SU(3)$-gluodynamics. Work in this direction is now in progress [28].

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